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Divide And Conquer

# Fake Coins

­­In order to solve this problem, my approach was the following:

* If we are down to 3 coins:
  + Balance 1st coin with 2nd coin.
  + Balance 2nd coin with 3rd coin.
  + If both sides are unbalanced, the fake is in the middle.
  + If the left side is unbalanced, the fake coin is the 1st.
  + If the right side is unbalanced, the fake coin is the 3rd.
* If we are down to 4 coins:
  + Balance 1st coin with 2nd coin.
  + Balance 3rd coin with 4th coin.
  + If the left side is unbalanced, use getFake3() with 1st, 2nd and 3rd coins.
  + If the right side is unbalanced, use getFake3() with 2nd, 3rd and 4th coins.
* If we are down to 5 coins:
  + Use getFake4() for 1st to 4th coins.
  + If the result is different from -1, we found the fake coin, return it.
  + Otherwise, the fake coin is the 5th coin.
* If we are down to 6 coins:
  + Balance the left side using getFake3() from 1st to 3rd.
  + Balance the right side using getFake3() from 4th to 6th.
  + Whichever result is different from -1 will be the fake coin.
* Else:
  + General approach: divide each part into halves and take balanceLeft and balanceRight on each half.
  + Problem: how to choose elements in order to always have the same amount of coins on each part of the balance?
  + Solution:
    - Distance = end – start
    - Center = distance / 2
    - For both variables, if the number is odd you do not repeat the central element when weighing. If the number is even, you repeat that element.
    - Odd distance, odd center 🡪 do not repeat central element on either half.
    - Odd distance, even center 🡪 repeat central element only on each half.
    - Even distance 🡪 repeat general central element.
    - Even distance, even center 🡪 repeat the central element on each half.

Complexity:

There will only be one recursive call in the whole algorithm, which means that a = 1. In each call, the vector size will be divided in half, which means that we have a Divide & Conquer by Division scheme with b = 2. There are no loops in the algorithm, only the one used by the balance to weigh the coins, a method which is O (n), so k = 1. Thus, the final complexity can be calculated using the formula:

a < bk 🡪 O (nk) = O (n)

Times table

|  |  |  |
| --- | --- | --- |
| Size | Total time | time(micros) |
| 1000,00 | 860,00 | 0,86 |
| 2000,00 | 1364,00 | 1,36 |
| 4000,00 | 2611,00 | 2,61 |
| 8000,00 | 559,00 | 5,59 |
| 16000,00 | 1064,00 | 10,64 |
| 32000,00 | 2113,00 | 21,13 |
| 64000,00 | 4173,00 | 41,73 |
| 128000,00 | 841,00 | 84,10 |
| 256000,00 | 1659,00 | 165,90 |
| 512000,00 | 3379,00 | 337,90 |
| 1024000,00 | 686,00 | 686,00 |
| 2048000,00 | 1564,00 | 1564,00 |
| 4096000,00 | 3275,00 | 3275,00 |
| 8192000,00 | 640,00 | 6400,00 |
| 16384000,00 | 1298,00 | 12980,00 |
| 32768000,00 | 2705,00 | 27050,00 |
| 65536000,00 | 614,00 | 61400,00 |

Times graph

The graph confirms what we knew: the algorithm is O (n) and thus solves the problem efficiently.